The Puzzle Universe

A history of maths

in more than 300 puzzles

Ivan Moscovich

LANNOO

'If you always do what interests you, at least one person is pleased'

– Katherine Hepburn

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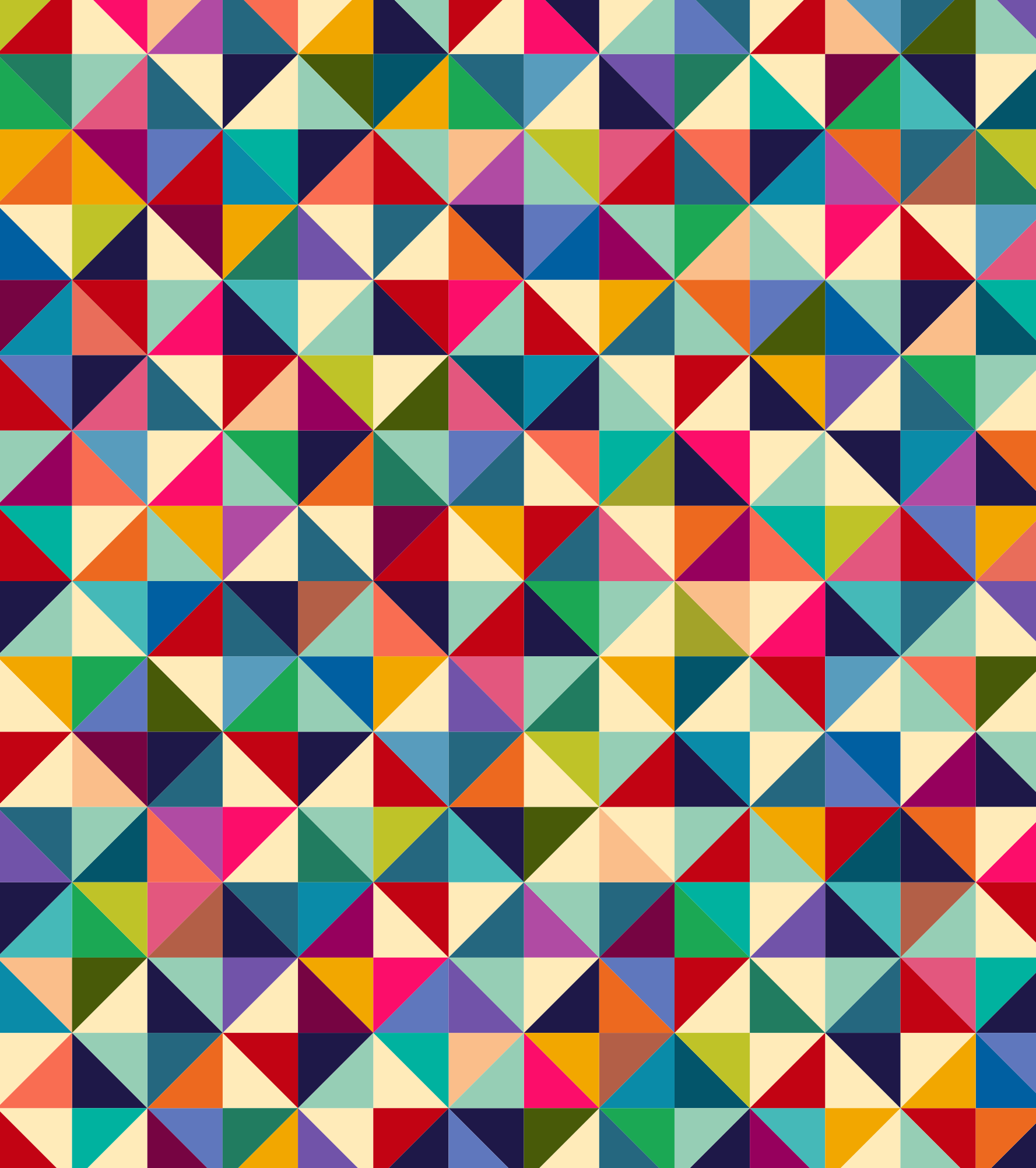


CHAPTER

1

THINKING ABOUT PLAYTHINKS AND YOUR BRAIN

CREATIVITY AND INTELLIGENCE

Creativity and creative people have been regarded with wonder and admiration throughout human history. They seem to be able to remain connected with that childlike state of wonder, and use it for fun, enjoyment, creativity, and much more.

How do they do it? How can we learn to be more creative? Creative people themselves can be a resource in answering this question.

Great scientists, artists and thinkers are highly motivated, challenge assumptions, recognize hidden patterns, see in new ways, make new connections and take advantage of chance.

Without creativity, human beings would have remained in a Paleolithic existence. Creativity is the most powerful mode of human thought and advancement. It is the resource that we all have to draw upon to enjoy and understand our lives and build our world.

There is no such thing as a recipe for creativity. It is also very difficult to define. It is more than just the process by which new ideas are generated. Creativity is really a different way of thinking, one that is preoccupied with fundamental relationships, arrangements and connections.

And, indeed, the more connections a creative mind can make, the more paths open up to finding a unique and satisfying answer to a problem. Psychologist Edward de Bono called this mindset lateral thinking, and it is commonly found not only in pioneering scientific minds but also in artists and other visionaries.

The ability to "think outside the box", to think differently, in a novel and unconventional way, is highly sought after today. We are entering a new age where creativity is becoming increasingly important.

Yet these creative people are not endowed with any special gift. For the first five years of life, every child is a creative thinker, with insatiable curiosity. Later, as we get older, we acquire mental blocks that obscure the essence of problems and often lead us away from even the most obvious solutions. We all have the potential to be creative; we just don't think creatively most of the time.

Eureka!

Innovation begins with creative ideas, requiring expertise. "Chance favors only the prepared mind", observed Louis Pasteur. The second component is imaginative thinking skills. In moments of creativity we see things in new ways, recognize patterns, make new connections – The Eureka Moment. A venturesome personality, continuously seeking new experience is the third component of innovation, followed by intrinsic motivation and the will to push through. Ideally, all of this is encouraged by a creative environment.

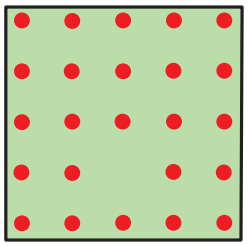
Most of us grew up with a concept of intelligence that is driven by tests: the person who can answer the most questions is thought to be the most intelligent. But imagining that intelligence can be boiled down to a single number – the IQ – is an obsolete notion. There have been a number of attempts to develop a Creativity Quotient for an individual, along the same lines as the Intelligence quotient (IQ), but they have been unsuccessful. Another flaw in early attitudes was the idea that intelligence is fixed at birth. Many late researches have shown that IQ scores can be significantly raised through appropriate training. According to Bernard Devlin, genes account for no more than 48 percent of a person's IQ, while 52 percent is a result of prenatal care, environment and education.

If you find yourself having difficulty with some of the puzzles, don't worry that you are not "smart" enough to do puzzles. It is all a matter of freeing up your latent creativity. With the proper mindset, anyone can do these puzzles.

And if you find the puzzles easy, congratulations!

"Creative thinking – in terms of idea creativity – is not a mystical talent. It is a skill that can be practised and nurtured."

– Edward de Bono, psychologist (Malta, 1933)



PEGBOARD SQUARES

How many different sizes of squares can you get by stretching rubber bands along four pegs of the pegboard?

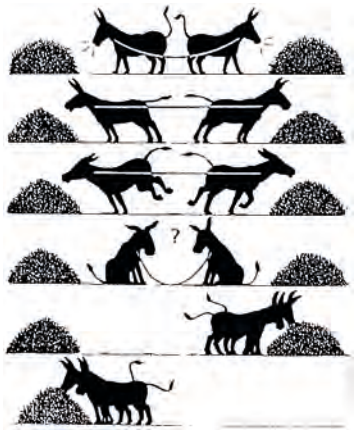
1 CHALLENGE ●●○○○○ REQUIRES COMPLETED ○ TIME

BLIND DATE PROBLEM

The computer dating website has set you up on a blind date. She will be waiting for you holding this book. Approaching the place you see Julia Roberts with the book. What's the problem?

2 CHALLENGE ●○○○○○ REQUIRES COMPLETED ○ TIME

CREATIVITY AND PROBLEM SOLVING



Problem solving can teach us a little about the way in which our brains function. Thinking can be hard work, hence the natural human tendency to do as little of it as possible. This is visible in the hit-and-run approach to problem solving: pick the first solution that comes to mind and run with it.

Your subconscious mind stores all of your previous life experiences, your beliefs, your memories, your skills, all situations you've been through and all images you've ever seen. It's time to remove consciousness from its privileged place in problem solving. The real power of creativity lies in the subconscious. As modern cognitive science has revealed and confrmed, our brains operate with a vast unconscious mind that even Freud never suspected. Much of our information processing occurs below the level of conscious awareness – offstage and out of sight. Our unconscious mind feeds our insight, creativity and intuition. It is fast, automatic, and effortless, while our conscious mind is deliberate, sequential and rational, requiring an effort to employ. Theories of Amos Tversky and Daniel Kahnemann indicate that humans have developed mental shortcuts, called heuristics, which enable quick and efficient judgments and actions. Experience teaches our intuition and many judgments become automatic (driving a car; chess masters who may have 50,000 chess patterns stored in their memories can play "blitz chess" picking the right moves intuitively, etc).

"Creativity comes from the abrasive juxtaposition of life experiences."

–Mario Capecchi, Nobel Prize winner for Medicine. His mother was taken to Dachau concentration camp. Mario, age 4, landed on the streets for 6 years, until they were reunited.

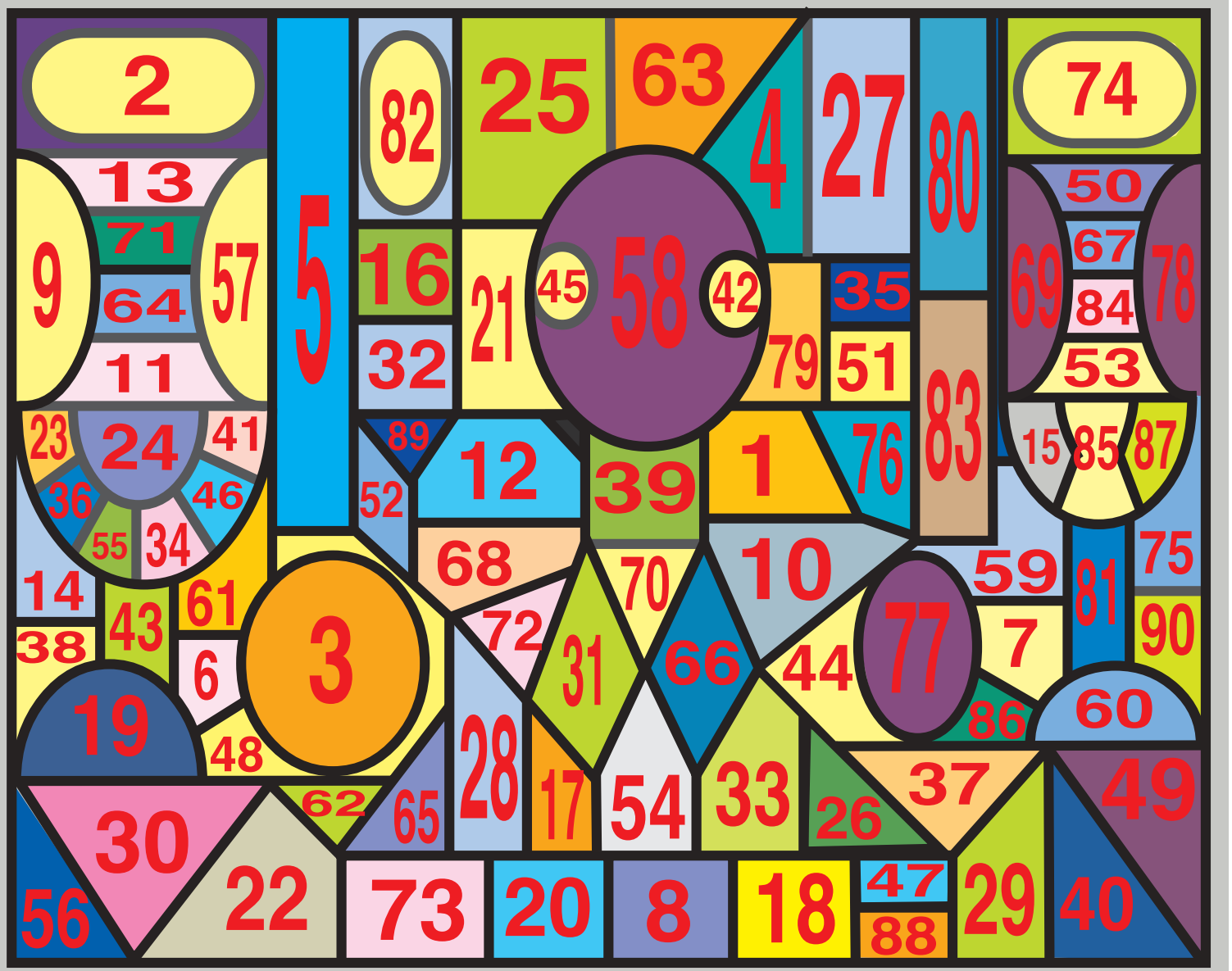
INTRODUCING YOUR SUBCONSCIOUS

"In the course of our lives, the subconscious handles over 90% of our problem solving. Complex decisions are best left to your unconscious mind to work out," according to recent research carried out by Prof. Ap Dijksterhuis at the University of Nijmegen. "Overthinking a problem could lead to expensive mistakes."

It suggests that the conscious mind should be trusted with simple decisions only.

Thinking hard about complex decisions based on multiple factors appears to confuse the conscious mind resulting in people concentrating only on a subset of information, often resulting in unsatisfactory decisions. In contrast, the unconscious mind appears to be more capable of considering the full information and therefore produce a more satisfactory decision.

Experiments demonstrate that we are all capable of such "nonconscious learning", as we shall experience from the two counting puzzles on page 14 and 15. Faced with complex decisions involving many factors, the best advice may be to take time out – to "sleep on it" – and to await the intuitive result of our unconscious processing. Today's cognitive science enhances our appreciation for intuition but also reminds us to always carefully check it against reality.



COUNTING FUN – TEST I

Two surprising visual tests may reveal how your subconscious works and solves problems, in this instance just by counting from 1 to 90.

Counting is the oldest mathematical activity of humankind and also one of the most powerful and fundamental ideas ever conceived by humans.

The idea that for each natural number, there is a next number, offered great mathematical advances.

This test and the test on the following page will give counting a new twist. The object in both is simply to find how long it will take you to find the consecutive numbers from 1 to 90 in succession, just by looking, without skipping any (consecutive) number or numbers, and also without marking the page as you search for the next consecutive number.

To complete the two tests, you must start by finding 1, then 2, 3, 4 and up to 90.

Cheating is of course not allowed and it would spoil the fun, believe me.

Repeat each test two or three times and mark your times in minutes for each count in the chart on the next page.

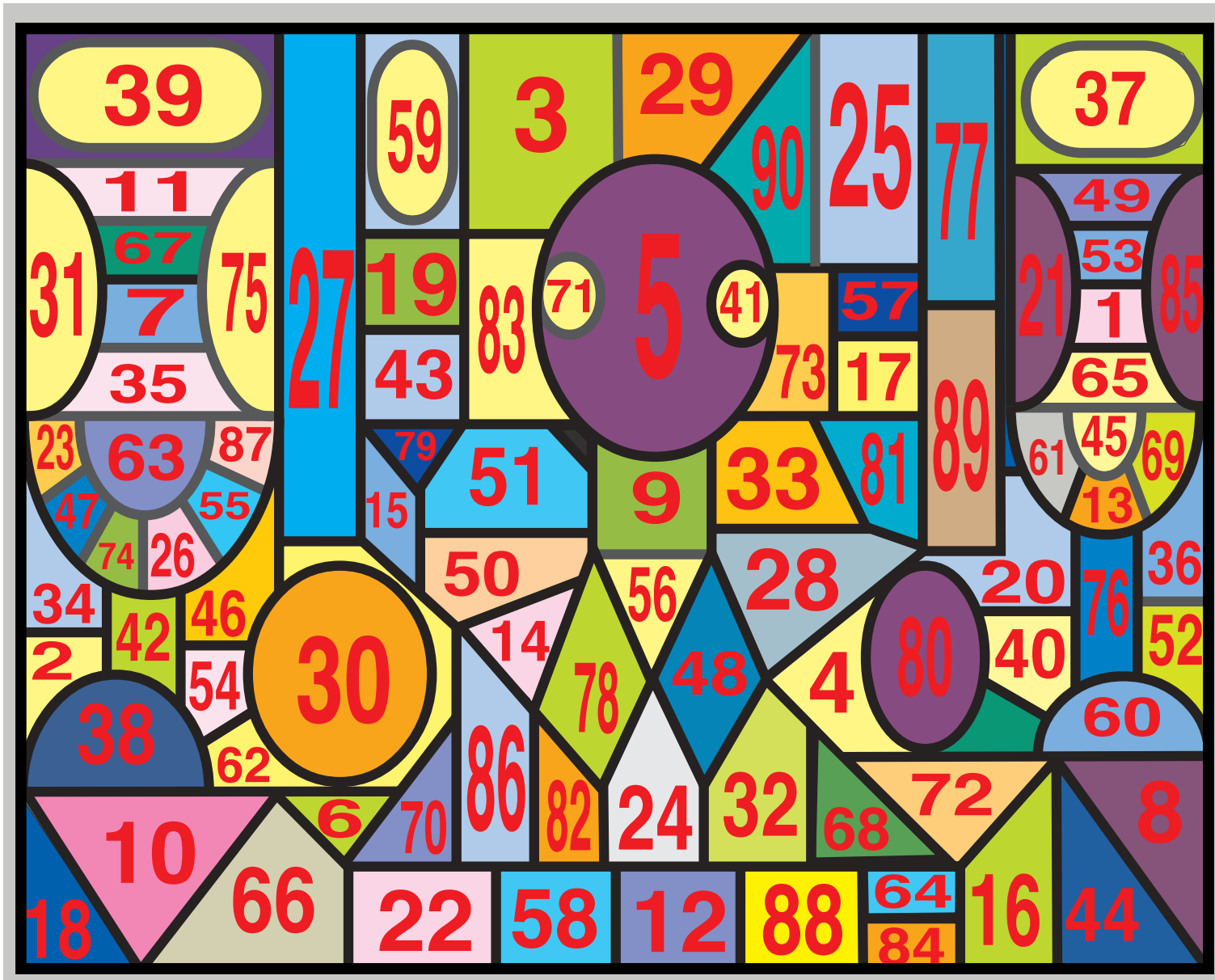
Your first surprise will be that the tests will take much longer than expected. You will also note that as you repeat the tests your times may improve. But the results of the second test will be even more surprising.

3

CHALLENGE ● ● ○ ○ ○ ○

REQUIRES

COMPLETED ○ TIME 88 88



COUNTING FUN – TEST 2

Repeat the two tests two or three times and mark your scores in the tables on the right.

It may surprise you that in test 2 there may be a very significant improvement in your scores compared to your results in test 1.

If this happens, can you find an explanation why such unexpected improvements happened?

YOUR SCORES

TEST 1	Score in minutes
first	
second	
third	

TEST 2	Score in minutes
first	
second	
third	

4

CHALLENGE ● ● ○ ○ ○ ○

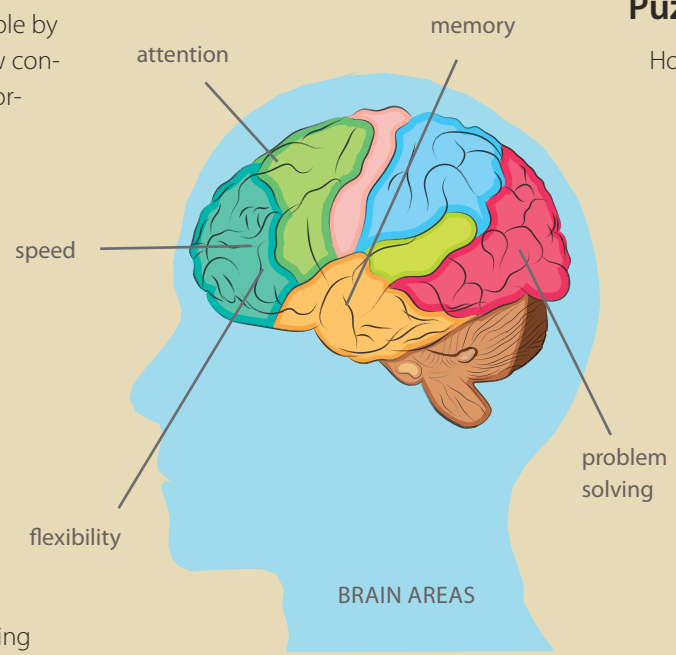
REQUIRES

COMPLETED ○ TIME

PUZZLES AND YOUR BRAIN

Puzzles are more than just an entertaining pastime

Solving a variety of different types of puzzles will improve your brain and ward off mental deterioration and age-related illnesses. The brain is a very complex machine that is constantly creating and reinforcing connections between its 100 billion cells. Exercising the brain, for example by solving puzzles, helps to create new connections and enhance mental performance in the long run. Memories are formed by these connections between brain cells, and by the capacity of each neuron to chemically signal the 10,000 or so other cells it connects to. Puzzles can support essential brain functions such as memory retrieval and the ability to process new information by strengthening the connections between brain cells. Engaging in problem solving exercises stimulates the creation of new connections in the brain whilst strengthening old ones.



Use it or lose it

If you don't give your brain a solid workout, your mental strength starts to deteriorate. As you age, it is important to keep your brain fit by playing a diversity of different types or puzzles.

Problems and puzzles can be simplistically divided into ones that require insight (a mental leap) and ones that are solved through a more systematic analysis. What is more common in problem solving: insight or analysis? Like most of these kinds of debates, it appears that both play an important role. The ability to switch between these two brain states is important, and if you try to solve problems that ask for both deep analysis as well as out-of-the box insight (as many types of puzzles do), this would add to your brain plasticity and health during the aging process.

Nowadays, brain scientists are asked whether doing puzzles helps stave off Alzheimer's and other forms of mental deterioration. Alzheimer's is a brain disease that causes increasing loss of memory and other mental abilities. It is the most common cause of severe memory loss in older adults. It attacks very few people before the age of 60, becoming increasingly common thereafter. In the not-too-distant past, older adults suffering from severe memory loss were often labeled "senile." But it is recognized today that they were probably suffering from Alzheimer's.

As a brain disease, Alzheimer's raises the question of whether or not puzzle activities can be used in helping to prevent it. Many studies strongly suggest that puzzle activities may indeed prevent mental deterioration and many organizations worldwide endorse puzzles as part of a preventive strategy.

Puzzle instinct

However, the literature on the correlation between puzzles and brain fitness is not extensive and, when looked at with a critical eye, really does not establish a definite correlation. The reason is that doing a certain genre of puzzle over and over, as most people are inclined to do, does not provide enough diversity for the brain. The brain does seem to need many different types of stimulating input in order to keep functioning. A great variety is needed.

In general, one can argue that the areas of the brain activated by puzzles of particular kinds might be larger than normal. This seems to be a logical assumption, but more studies are needed.

Studies by Marcel Danesi at the University of Toronto revealed that we do indeed all have different puzzle preferences and problem solving abilities. Some people only love crossword puzzles, others just like logic ones (like Sudoku). A few enjoy a mixture of them. In some of Danesi's experiments, puzzle materials that fell into "unpreferred" categories were given to groups of students. After a period of 8 months a significant number (around 74%) claimed that they started to like the puzzle genre they once disliked. By simply doing puzzles, our "puzzle instinct" seems to kick in and allow us to enjoy all genres.

PUZZLE BASICS

We continue with a random sample of 24 typical classic thinking puzzles from my colllection of about 5000 classic puzzles to warm up your puzzle-solving ability. They are not difficult and don't require any previous mathematical knowledge. They clearly show a diversity of types of puzzles, including a wide range of mathematical, logical ideas and basic principles.

5

CHALLENGE ● ● ● ○ ○ ○

REQUIRES

COMPLETED ○ TIME 00 00

SNAIL'S PACE

A little snail climbs up a window 90 cm high. If every day it climbed up 11 cm, and every night it dropped back 6 cm, how many days without stopping did the little snail travel to reach the top?

6

CHALLENGE ● ● ● ○ ○ ○

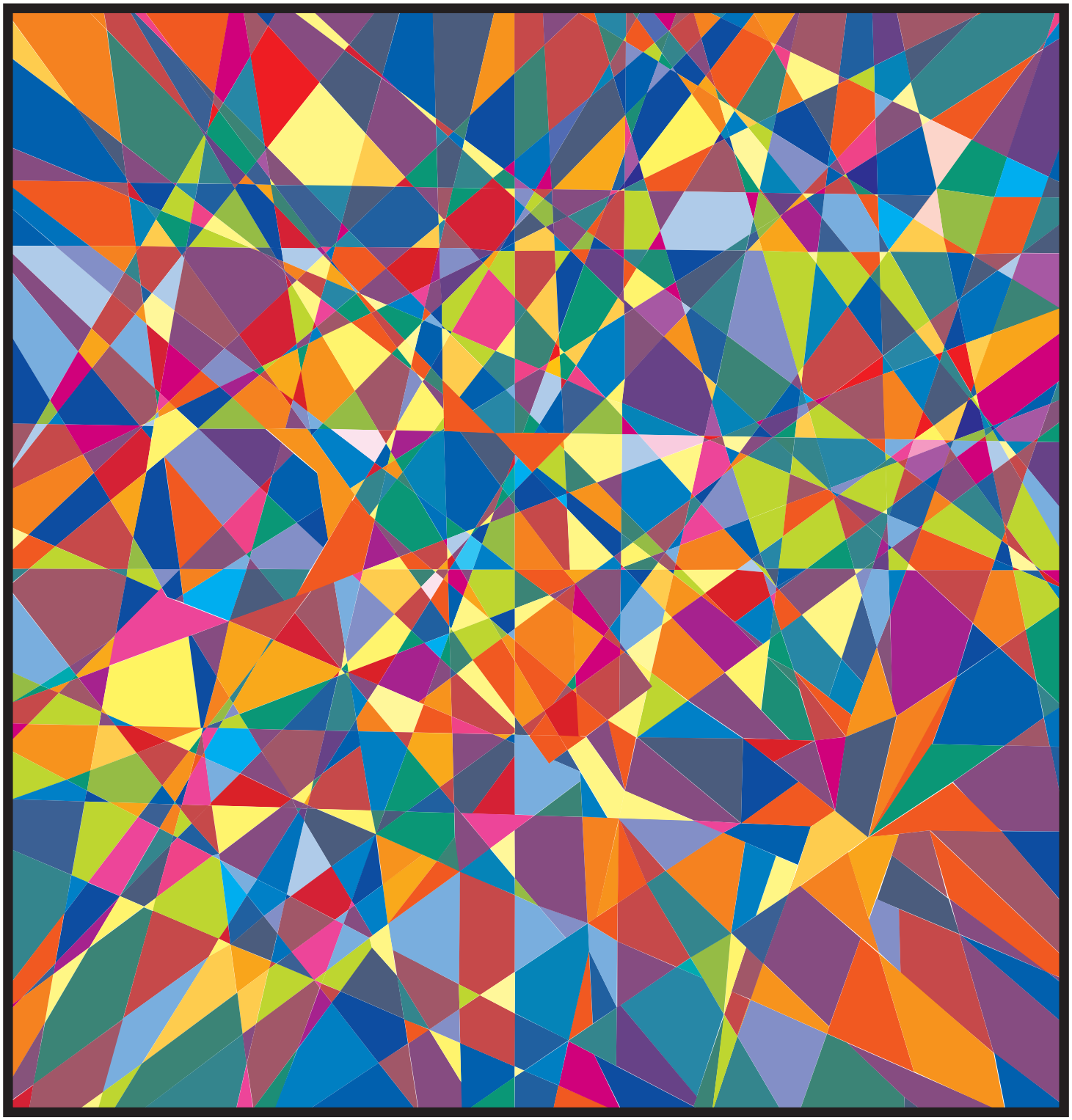
REQUIRES

COMPLETED ○ TIME

A HUNDRED TOTAL

A very old classic arithmetic problem is the one using consecutive digits from 1 to 9 and inserting mathematical symbols between them to form the sum total of 100. There are endless variations of the problem, including the one in which only plus or minus signs are allowed. Martin Gardner demonstrated the solution using the maximum number of plus signs and the minimum number of minus signs.

Can you insert plus or minus signs in the above number sequence to achieve such a solution? Hint: two of the consecutive digits will form a two-digit number. There may be other solutions using more than just one two-digit number, and/or more than just minus and plus signs. How many solutions can you find for this problem?



7

CHALLENGE

REQUIRES

COMPLETED

TIME

HIDING REGULAR POLYGONS AND A STAR

How long will it take you to find the outlines of seven regular polygons and a ten-pointed regular star?

8

CHALLENGE

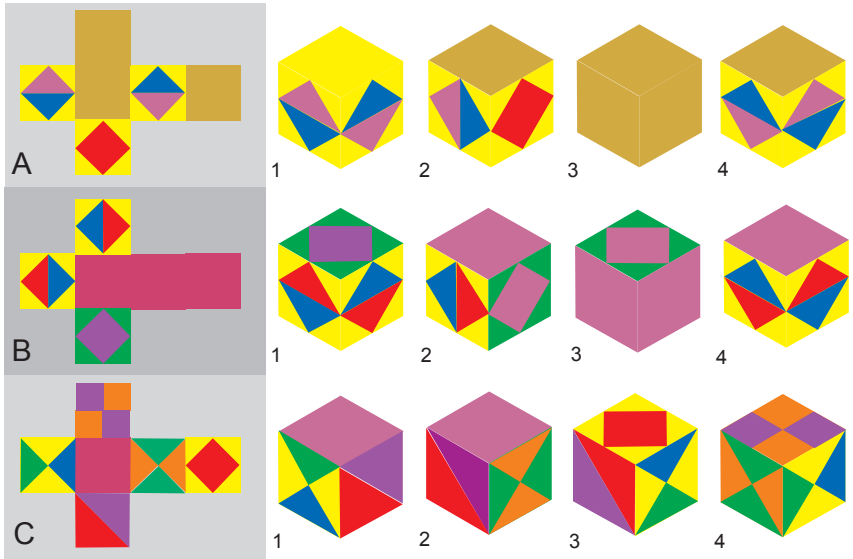
REQUIRES

COMPLETED

TIME

BIRDS ON A WIRE

Imagine a very long wire on which a multitude of birds are randomly distributed looking one way or another along the wire at their nearest neighbor. Can you guess how many birds will be observed by one, two, or none of their neighbors if there is an infinite number of birds on the wire? In the example above only 72 birds were randomly distributed, so your guess will be an approximation.



9 CHALLENGE ● ● ○ ○ ○ ○
REQUIRES
COMPLETED ○ TIME

CUBE FOLDS

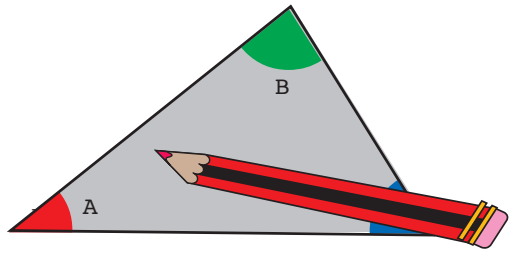
On the left are unfolded nets of cubes with designs on their faces (A-B-C). On the right are four isometric drawings of cubes. The object in each case is to match up each unfolded cube with the correct isometric cube.

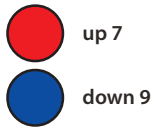
10 CHALLENGE ● ● ● ○ ○ ○
REQUIRES
COMPLETED ○ TIME

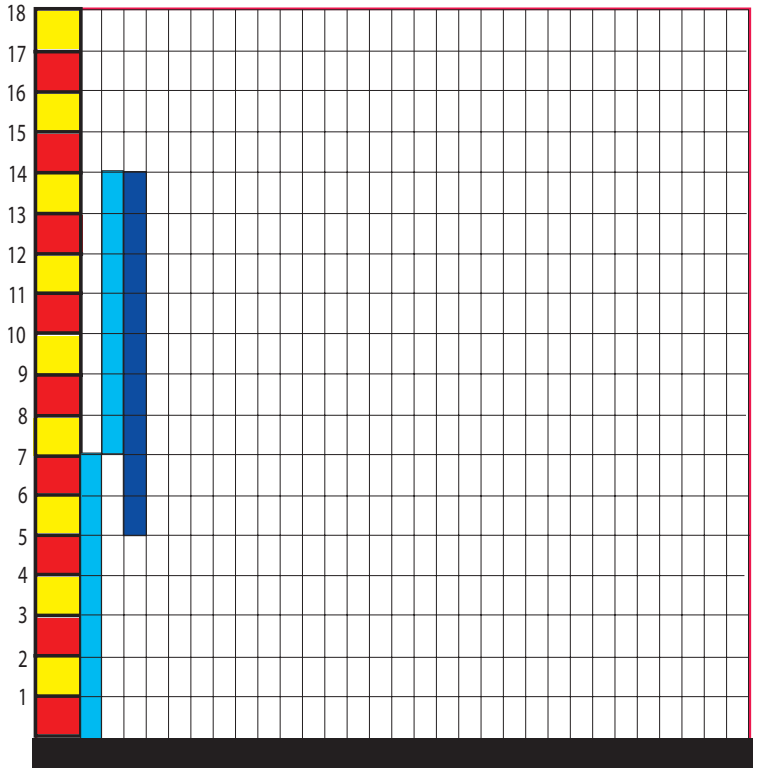
ANGLES IN A TRIANGLE

Euclid proved that the three angles of a triangle add up to a straight angle (180 degrees). The beauty of mathematics is that often, amateurs with a bit of insight can make new discoveries and come up with new evidence.

Luther Washington, today a mathematician at Stanford, as a very young student had an idea to prove it in a simpler way, using only a pencil. Can you work out how he did it?



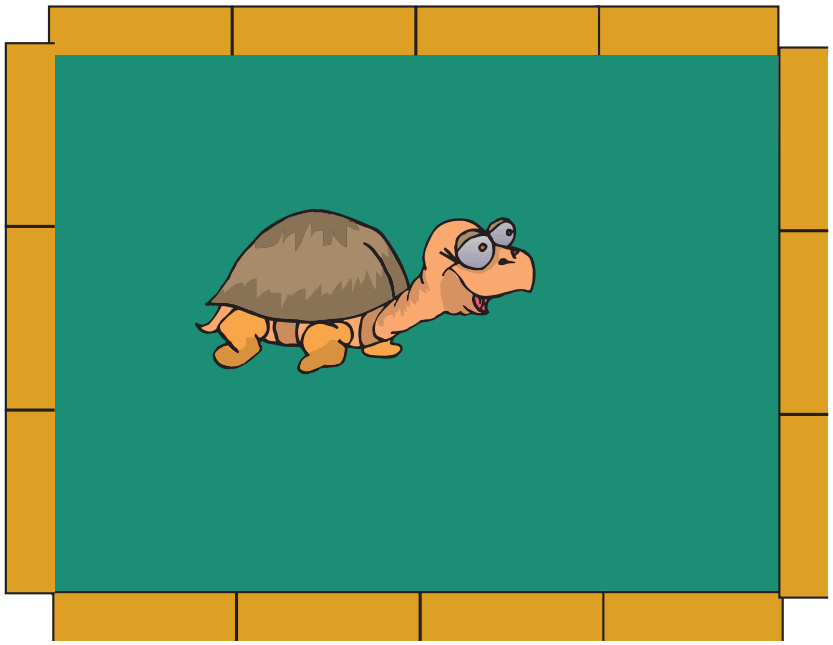




11 CHALLENGE ● ● ○ ○ ○ ○
REQUIRES
COMPLETED ○ TIME

ELEVATOR GOING UP AND DOWN

In an 18-floor building there is only one strange elevator which only has two buttons: one 'up' and the other 'down'. The up button takes you up 7 floors (or doesn't move at all if there are no floors available), and the down button takes you down 9 floors (or doesn't move if there are no floors available). Is it possible to get from the ground floor to any floor by taking the elevator? How many times will the maintenance man have to push the buttons to get from the ground floor to all the other floors and in what sequence will he visit the floors? The first three trips are demonstrated.

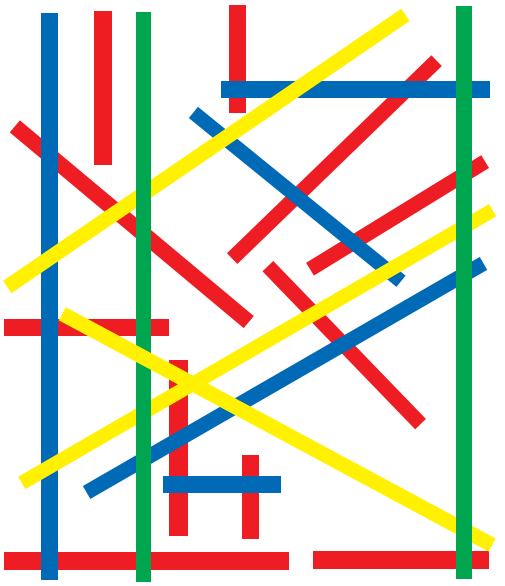


ENCLOSURE

Tim found 14 bricks to make a fence in the garden for his new pet, a small turtle. Since his turtle has grown over the years, Tim wants to enlarge the enclosure as much as possible, using the same number of bricks. How can he do this?

12

CHALLENGE ● ● ○ ○ ○ ○
REQUIRES
COMPLETED ○ TIME

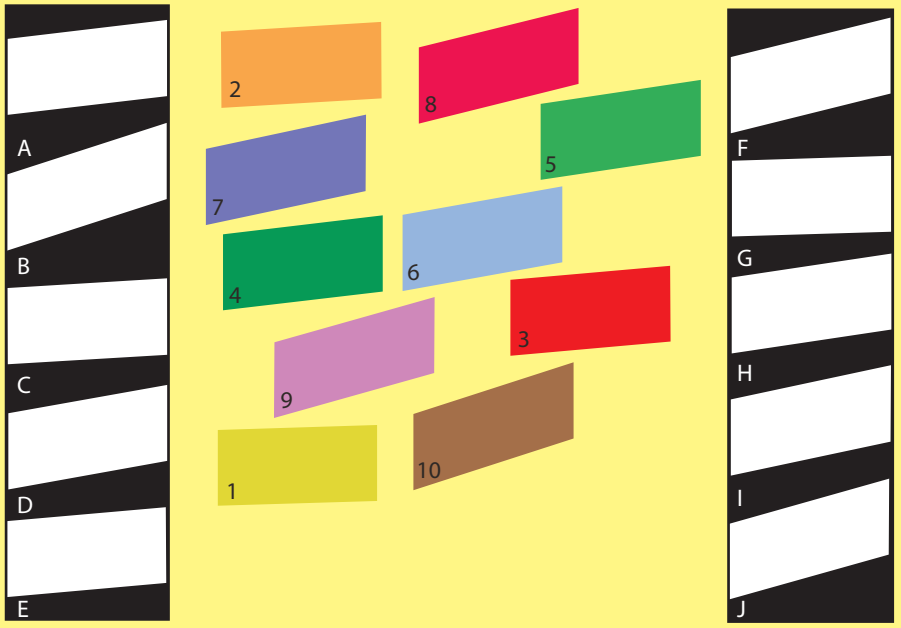


PLAYGROUND

You are looking from above on a playground on which planks are piled one on top of another. Can you identify the highest point?

13

CHALLENGE ● ● ○ ○ ○ ○
REQUIRES
COMPLETED ○ TIME



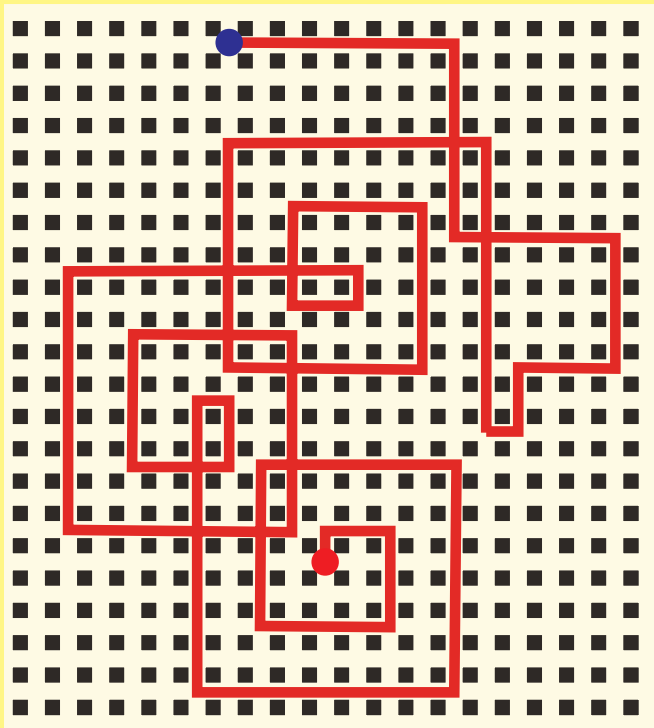
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	

FILLING GAPS

Just by looking, fit the numbered colored shapes into their corresponding gaps and pair them with the lettered gaps in the accompanying chart. How many mistakes did you make? You may be surprised!

14

CHALLENGE ● ● ○ ○ ○ ○
REQUIRES
COMPLETED ○ TIME



MOLE WALKS

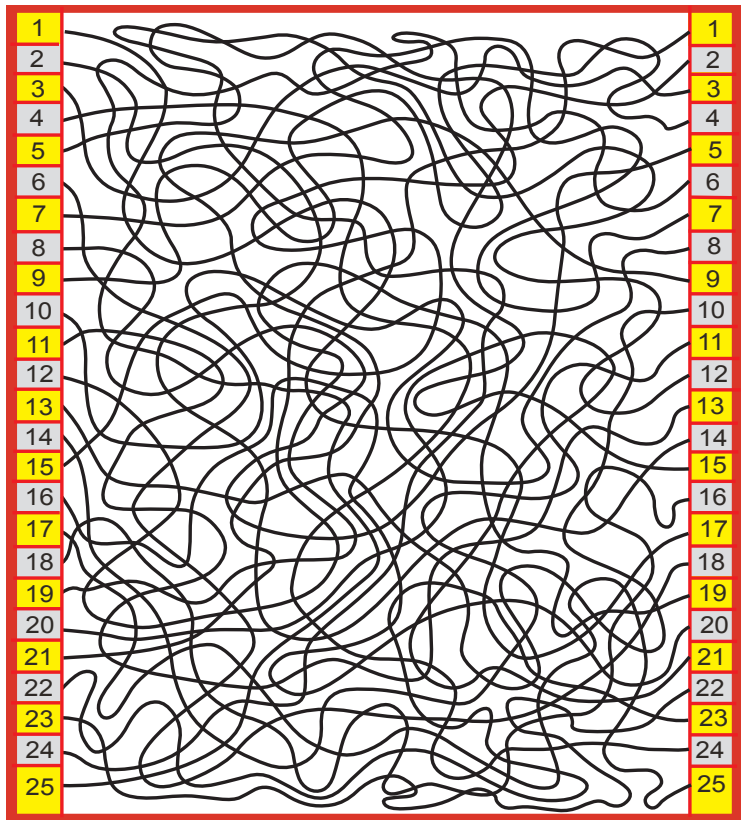
The mole starts from the red point. The red line shows its path until it ends at the blue point. Can you work out the logic of its path up till the point where it was forced to change the rule? At which point did this happen?

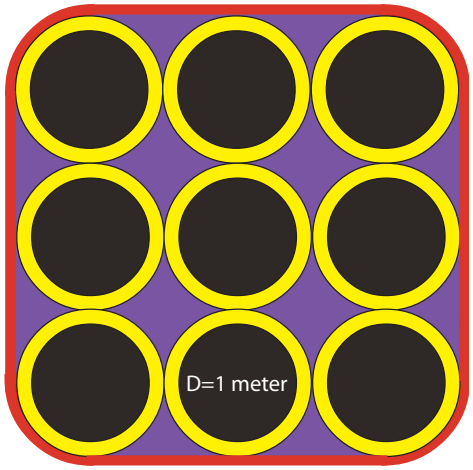
15 CHALLENGE ● ● ○ ○ ○ ○ REQUIRES COMPLETED ○ TIME

LINE UP

How many lines will you be able to trace through just by looking, before you lose track? Stability of attention is the ability to direct it towards something for a long period of time.

16 CHALLENGE ● ● ○ ○ ○ ○ REQUIRES COMPLETED ○ TIME





PIPES

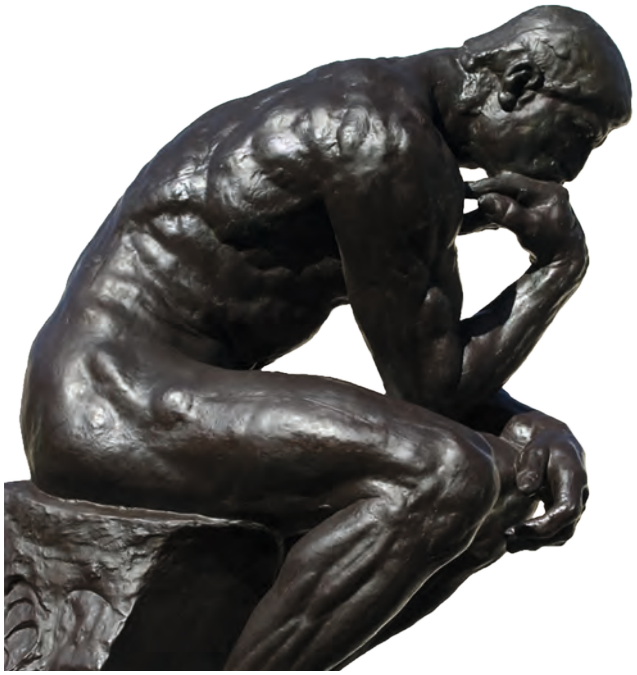
Nine pipes are tightly tied together by a red metal band. How long is the red band?

17 CHALLENGE ● ● ● ○ ○ ○ REQUIRES COMPLETED ○ TIME

SWEETS

A piece of cake and an ice-cream cost two and a half dollars between them, but the cake costs a dollar more than the ice-cream. How much does each cost?

18 CHALLENGE ● ● ○ ○ ○ ○ REQUIRES COMPLETED ○ TIME



PATTERNS OF MIND

The ancient Greek "deiknymi", or thought experiment, was the most ancient pattern of mathematical proof. Thought experiments are ideas of imagination used to investigate the nature of the unknown, often resembling riddles. The common features of thought experiments include visualization of a hypothetical scenario, experimentation and conceptualization of what is happening.

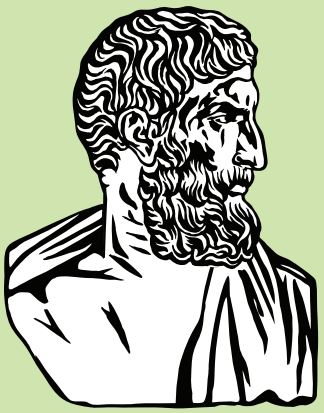
The idea and simple reasoning behind thought experiments is that just by the power of thinking we can discover new things about the world, which early on created great interest in philosophy. Many famous thought experiments played an enormous role in advancing mathematics and science, like Einstein's elevator, Newton's apple, Schrödinger's cat, Maxwell's demon, Newton's satellite principle, Galileo's balls, and many other.

The infinite universe

One of the most beautiful early thought experiments in its simplicity and elegance is the "Infinite Space" thought experiment of Archytas and Epicurus. Archytas believed that the universe is infinite and has no limits. In his thought experiment he used the idea that someone is at the border of the universe and stretches his hands beyond this limit. About a hundred years later, Epicurus, in a similar thought experiment, imagined an arrow flying endlessly through space without encountering any obstacle, thus proving that space is infinite. On the other hand, if it bounces back encountering an obstacle like a wall, at the "end" of space, it would again prove that space is infinite, since there must be something behind the wall.

The ideas of Archytas and Epicurus are beautifully visualized in the Flammarion engraving from 1888, believed to have been originally created in the 16th century.

Plato and Aristotle did not accept the idea of an infinite universe because they had problems accepting the idea of the infinite in general. Up to medieval times Aristotelian cosmology was accepted in terms of which space was finite and had a definite edge.



EPICURUS (341-270 BC)

The philosophy of Epicurus was to make a happy life for everyone with more pleasure and less sadness. According to him, the best way to achieve this objective is not to want anything. Epicurus's philosophy had an important influence on early Christianity. His Infinite Space thought experiment is a beautiful and important classic.

ARCHYTAS (428-347 BC)

Ancient Greek mathematician, scientist of the Pythagorean school and one of the founders of mathematical mechanics. Among his many achievements he constructed a wooden pigeon, the earliest self-propelled flying device (probably by steam) capable of flying some 200 meters.